Catastrophe Risk Management
– Implications of Default Risk and Basis Risk

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Abstract

A major problem for insuring catastrophic risk is that, as a disaster causes damages to many insureds at the same time, such insurance and in particular reinsurance contracts are often subject to considerable default risk. On the other hand, the securitization of insurance risk, for example via a catastrophe bond, can be designed to completely avoid default risk. In many cases, however, the payout from an insurance-linked security is tied to some stochastic variable, an index, which is correlated, but not identical, with the insured's actual losses. Therefore, such an instrument will usually not provide a perfect hedge. There will be some mismatch, the so-called basis risk. This paper investigates how the trade off between default respectively credit risk and basis risk affects optimal risk management solutions, when (re)insurance and risk securitization are used simultaneously. In particular, the impact of credit risk and risk securitization on the optimal reinsurance contract is analysed.

Keywords: Insurance, Financial Markets, Alternative Risk Transfer, Decision Making and Risk

JEL-Classification: G10, G22, D81

1 Introduction

The temporary shortage of catastrophe reinsurance in the early 1990s, in particular following hurricane Andrew,[1] set off a search for alternative risk transfer (ART) solutions. The focus was primarily on tools that would enable a direct transfer of risk using the financial markets, via insurance-linked securities.[2] Capital market insurance solutions could be observed since 1992.[3] Over the past few years, transfer of insurance risk via the financial markets has mainly been carried out using over-the-counter securities, such as catastrophe bonds (cat bonds).

A cat bond is a bond in which the interest and/or – depending on the specific design – the principal is (partially) forgiven when a pre-defined catastrophic event occurs. The typical structure of a cat bond issue is as follows:[4] A special purpose vehicle (SPV) is set up, usually as an offshore reinsurer, its purpose solely being the handling of that specific securitization. The SPV reinsures the primary and backs up this contract through the issuance of the cat bond. The principal invested is held in trust. If no loss occurs, principal and interest are paid back to the investors, whereas in case of a loss this amount is reduced by the coverage that goes to the primary.[5]

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[1] See, in particular, *Froot* (2001), p. 540.

[2] To motivate the interest in financial market solutions for the transfer of insurance risk, authors often refer to the size of the financial markets or their daily fluctuations in comparison to the size of a major natural catastrophe (see, e.g., *Durbin*, 2001, p. 305, *Laster* and *Raturi*, 2001, p. 13, or *Durrer*, 1996, pp. 4-5). For example, a USD 250 billion event would only represent less than 0.5% of the total market value of publicly traded stocks and bonds of USD 60 trillion (*Laster* and *Raturi*, 2001, p. 13).

[3] The total volume of transactions carried out since then exceeds USD 13 billion (*Munich Re ART Solutions*, 2001, p. 11). Compared to the size of the reinsurance market, this is still not very significant. For example, the catastrophe excess of loss coverage purchased in the worldwide reinsurance market in the year 2000 amounted to CHF 107 billion (*Durbin*, 2001, p. 301). The at first rapid increase in the use of financial catastrophe risk management tools halted in the late 1990s after a decrease in reinsurance prices (see *Laster* and *Raturi*, 2001, p. 18). Particularly the consequences of September 11th on reinsurance capacity and pricing, however, might cause the growth of the market for insurance-linked securities gain speed again.

[4] See, e.g., *Belonsky*, *Laster*, and *Durbin* (1999), p. 5.

[5] For a more comprehensive discussion of insurance risk securitization design possibilities as well as for data concerning recent transactions in this field see, e.g., *Durrer* (1996), *Baur* and *Schanz* (1999), *Belonsky*, *Laster*, and *Durbin* (1999), and *Laster* and *Raturi* (2001).

Cat bonds have had the biggest market share among recent insurance risk securitization transactions.[6] These bonds are mainly used by primary insurers and reinsurers to substitute or supplement traditional reinsurance or retrocession. It has to be emphasized, however, that such instruments can, of course, also be attractive risk management tools for companies from other lines of business. As an example, reference can be made to the cat bond hedging earthquake risk that was issued by Tokyo Disneyland in 1997.

One economic rationale for the attractiveness of certain kinds of risk transfer through the financial markets is that, in contrast to traditional (re)insurance products, these instruments can be designed in such a way as to avoid – or substantially reduce – *default risk* (credit risk).[7] For instance, the capital invested in a cat bond is provided ex ante and is therefore in any case available when a catastrophe occurs that triggers the coverage.[8, 9]

This is an important feature since, in particular, natural disaster hazards often impose a significant insolvency risk for reinsurance companies active in that business, implying that their contracts are subject to default risk. The problem mainly arises from the potential of a regional accumulation of losses as it is typically incurred by catastrophic events. The threat of loss accumulation leads to high correlation between the different local primaries' portfolios and, therefore, between claims from different contracts in a reinsurer's portfolio. For the single primary insurer, this leads to an increased default risk with respect to catastrophe reinsurance.[10]

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6 See *Laster* and *Raturi* (2001), p. 19.

7 See, e.g., *Croson* and *Kunreuther* (2000), pp. 30-31, *Laster* and *Raturi* (2001), p. 14.

8 More precisely, the funds are usually invested in investment-grade securities and guaranteed by a highly rated company (see, e.g., *Laster* and *Raturi*, 2001, p. 14). This implies that contingent capital generated through issuance of a cat bond is – if at all – subject only to very little default risk.

9 The use of catastrophe options also avoids default risk to a great extent, as usually obligations are guaranteed by the exchange (see, e.g., *Laster* and *Raturi*, 2001, p. 18).

10 An illustrative example for the realization of default risk was hurricane Andrew which lead to a number of insolvencies in the reinsurance market. The following years were also characterized by a massive drop of the number of reinsurance companies due to a series of mergers and acquisitions (see *Holzheu* and *Lechner*, 1998). Considering that major factors determining a reinsurer's risk of insolvency are its worldwide spread and financial strength, this tendency of consolidation might – among other issues – also be a consequence of a growing awareness of default risk. See also *Laster* and *Raturi* (2001), p. 14: That default risk is an issue in reinsurance contracting is also reflected by market shares. In 1999, for example, among the world's 100 biggest reinsurance companies, only 20% of premiums were written by companies rated (by *Standard* & *Poor's*) below AA.

While default risk can be reduced or avoided through certain risk securitization instruments, this advantage is often tempered by another typical feature of such tools: In many risk securitization transactions the coverage does not directly depend on the hedging insurer's actual losses but on some other random variable which is correlated with the losses. For instance, the contingent payment from a cat bond can be tied to a (regionally defined) market loss index, or it can be based upon technical parameters describing the intensity of a catastrophic event (parametric trigger). Examples for the latter kind of an underlying are the Richter scale reading of an earthquake or the strength of a hurricane, observed in a certain region over a certain period specified by the contract.

The main advantage of market indexes or parametric triggers, besides their contribution to alleviate standardization, is the fact that, compared with reinsurance, they are largely or even completely out of the primary's control.[11] Therefore, the use of these underlying random variables offers an instrument to address *moral hazard*, which is a problem in almost every insurance relationship. It can be observed in primary insurance, but also in the relationship between a primary insurer and its reinsurer: A primary is in charge of risk selection and monitoring as well as settling losses with its customers. Considering the fact that it would normally be impossible or prohibitively expensive for the reinsurer to monitor these activities, reinsurance relationships will usually be characterized by asymmetric information. As a consequence, a primary's carefulness can be expected to decrease in the amount of its reinsurance coverage.

If, on the other hand, the trigger is a market loss index, moral hazard is limited to the primary's contribution to the index. By making use of a parametric trigger the moral hazard problem can even be avoided. However, the reduction or elimination of moral hazard incurs a certain cost. Typically, the less the underlying random variable can be influenced by the primary, the less useful is the contingent coverage as a hedging vehicle. The resulting mismatch between the loss and the coverage is called *basis risk*. For instance, the payment from a cat bond might not be triggered by an earthquake, since its strength is too low, even though substantial damages are caused in the primary's portfolio. On the other hand, a

[11] A parametric trigger has the additional advantage that the relevant numbers are usually available very quickly. Contrasting this, a market index typically needs a long time until it is fully developed, in particular due to time-consuming problems of loss-settling.

realization of basis risk could be that coverage from the cat bond is actually paid to the hedging primary although no significant individual losses are observed from that particular event.

This paper will analyze the way in which default risk (respectively, credit risk) and the combined occurrence of default risk and basis risk affect risk management decisions. Of particular interest is the impact of these risk components on the structure of optimal reinsurance arrangements.

So far, the demand for catastrophe coverage, provided through the new financial instruments, has been addressed by means of formal analyses only in a few papers: For example, *Doherty* and *Mahul* (2001) and *Doherty* and *Richter* (2001), investigate the trade-off between moral hazard and basis risk, when insurance can be used to insure the basis risk. It is shown that combining the two hedging tools might extend the possibility set and therefore lead to efficiency gains.[12] While these papers are concerned with an insurance coverage that is based upon an index-linked product, *Nell* and *Richter* (2002) consider optimal risk management solutions for the case where both tools can be used independently. They study the trade-off between the implicit transaction cost incurred by a reinsurer's risk aversion and the basis risk of a cat bond. In the following, a model will be used which modifies the framework introduced by *Nell* and *Richter* (2002) by incorporating default risk.

The impact of default risk on the demand for insurance, or respectively, on optimal insurance contracting has been addressed by *Schlesinger* and *Schulenburg* (1987) and *Doherty* and *Schlesinger* (1990). They show that certain important results from insurance demand theory do not hold when default risk is taken into account.

In a recent paper on this topic, *Cummins* and *Mahul* (2000) consider an insurance product that is subject to default risk as well as basis risk, since the insurer's payment is tied to an exogenous index. As mentioned above, the interaction between these two factors is also the topic of this paper. The focus, however, is different here, since, in contrast to *Cummins* and

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[12] As the combined product could easily be replicated by a reinsurance company that would use an index trigger in addition to the usual indemnity trigger (i.e. the primary insurer's actual loss), these results can also be interpreted as a product design recommendation for reinsurers.

Mahul (2000), a situation is considered with two different instruments (using different triggers): Insurance, on the one hand, is subject to default risk but can be used to generate a perfect hedge. Risk securitization, on the other hand, comes without default risk but incurs basis risk.

The remainder of the paper is organized as follows: The model is introduced in the next section. Section 3 presents the results, and section 4 briefly summarizes the main findings.

2 The Model

Consider the risk management decisions of a primary insurer with an initial risk situation $\tilde{X}$. Assume that the (nonnegative) random variable $\tilde{X}$ is continuously distributed with density function $f(x)$. The primary insurer is risk-averse and maximizes its expected utility. u_1 denotes the (three times continuously differentiable) primary's utility function ($u_1' > 0$, $u_1'' < 0$).[13]

The primary insurer can purchase traditional reinsurance or index-linked coverage or both. Let us first introduce reinsurance: A reinsurance contract will be characterized by an indemnity function, $I(x)$, that assigns the amount of indemnity to the realization x of the random variable $\tilde{X}$, the premium, Π, and the reinsurer's credit risk. The latter is the risk that the indemnity from the reinsurance contract would not be available when a loss occurs. It is expressed through the function of conditional probabilities that the reinsurer goes bankrupt and therefore cannot compensate the primary, given the amount of the primary's actual loss (x): These probabilities are denoted by $1 - p_R(x)$. Thus, $p_R(x)$ is the probability that the primary receives the coverage from the reinsurance contract, given x.

We assume that the function $p_R(x)$ is continuously differentiable and decreasing in x ($p_R'(x) \leq 0$). This assumption reflects that reinsurance default risk is typically correlated with the individual primary's actual losses from catastrophic events. Consider the example of a Californian primary insurer with a regionally concentrated book of business, wanting

[13] For a discussion of adequate assumptions concerning risk attitudes in entrepreneurial decision-making, see, among many others, *Greenwald* and *Stiglitz* (1990), *Dionne* and *Doherty* (1993), or *Nell* and *Richter* (1996).

to primarily reinsure the catastrophic consequences of a major earthquake. The primary would have to take into account that the likelihood of the reinsurer actually covering losses in accordance with the contract would, at least, not be increasing in the primary's actual losses. This is due to the risk of an accumulation of insured losses in the reinsurer's portfolio: The primary's individual losses from catastrophic events would be highly correlated with the losses of other primaries, particularly in the same local market. In case of a catastrophe the reinsurer's capacity, thus, would at the same time be needed for a multitude of its customers.

Clearly, the extent of credit risk, and in particular the shape of the function $p_R(x)$, crucially depends on the degree of regional diversification in the reinsured portfolio.[14] The limiting case is that the reinsurance company has only very little exposure to catastrophic loss in a certain area, such that here the probability of default is constant in x ($p'_R(x) = 0 \ \forall x$).

As the interdependence between the individual primary's losses and default risk is certainly characteristic for most catastrophe reinsurance relationships, it is interesting to analyze the impact the resulting correlation has on the optimal catastrophe reinsurance contract structure. The model frameworks used by *Schlesinger* and *Schulenburg* (1987) and *Doherty* and *Schlesinger* (1990) assume two-point loss distributions, and, therefore, cannot be employed to derive results concerning how credit risk affects the optimal indemnity scheme. So, with respect to the mere insurance demand theory point of view, the more general approach chosen here does not only allow for a generalization of certain results. It also enables an analysis of specific aspects of catastrophe (re)insurance demand. That – of course – will just be the first step, as we are primarily interested in the interaction between index-linked coverage and reinsurance.

Assume that all actors in the market possess complete information, implying there are no moral hazard problems. Furthermore, to keep the analysis as simple as possible, we will discuss the trade-off between basis risk and credit risk under the assumption that

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[14] Obviously, a reinsurance company's bankruptcy risk also depends on other factors besides the structure of its insured portfolio. In particular, the size of the reinsurer's own capital funds and its own reinsurance or retrocession policy are important determinants. In the model, these aspects would affect the shape of the function $p_R(x)$ and in particular the component of credit risk which is "independent" of $\widetilde{X}$ $(1 - p_R(0))$.

reinsurance incurs no transaction costs. In particular this means that the reinsurer is assumed to be risk-neutral.

Under these assumptions, the reinsurance premium in a competitive market can be calculated as follows:

(1) $$\Pi = \int_0^\infty p_R(x) I(x) f(x) dx.$$

The price of reinsurance equals the expected value of the random variable $I(\widetilde{X})$, corrected by a reduction that reflects default risk.

The reinsurance contract can be supplemented or substituted through index-linked coverage $H > 0$, which is triggered if an exogenous index $\widetilde{Y}$ reaches or exceeds a certain level $\bar{y}$ (since this kind of product is usually defined discretely, we concentrate – without major loss of generality – on the simple case of a stochastic variable with only the two possible outcomes 0 and *H*). The correlation between $\widetilde{Y}$ and $\widetilde{X}$ is expressed by the function of conditional probabilities

(2) $$p_H(x) := P\{\widetilde{Y} \geq \bar{y} \mid \widetilde{X} = x\}$$

(if a certain outcome is not specified, we also write $p_H(\widetilde{X})$).

Let $\bar{p}_H = E[p_H(\widetilde{X})]$. ($E[\cdot]$ denotes the expectation with regard to the distribution of $\widetilde{X}$.)

In the absence of transaction costs the index-linked product is sold in a competitive market at a rate that equals the expected payment, $\bar{p}_H H$.

Assume that $p_H(x)$ vanishes for sufficiently small *x,* and that $p_H(x) = 1$ for sufficiently large losses, and finally that there is an area where the trigger probability is strictly between 0 and 1 and increasing. To formalize this, we say that potential levels of loss x_1 and x_2 ($x_1 < x_2$) exist such that[15]

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[15] For a more detailed motivation of this approach to modeling the structure of a cat bond, see *Nell* and *Richter* (2002).

(3)
$$\begin{array}{ll} p_H(x) = 0 & x \le x_1 \\ 0 < p_H(x) < 1 & x_1 < x < x_2 \\ p_H(x) = 1 & x \ge x_2. \end{array}$$

The intuition behind this is as follows: If the primary is only hit by a very small amount of losses from its portfolio, it is highly unlikely that a triggering event occurred. Given, the primary's actual losses are even below x_1, the likelihood that the cat bond was triggered, is equal to 0. The conditional trigger probability increases in the amount of actual losses between x_1 and x_2, and finally, the assumptions mean that extremely high individual losses would only be observed if also a triggering event happens: Given the information that the primary's individual losses exceed x_2, the probability of a triggering event is equal to 1.

The function $p_H(x)$ is assumed to be differentiable with:

(4) $p'_H(x) > 0 \quad \text{for } x_1 < x < x_2.$

3 Results

The primary's optimization problem in the model framework introduced in section 2, is as follows (where W_1 denotes the initial wealth):

(5)
$$\begin{aligned} \max_{I(\cdot),H,\Pi} U := \int_0^\infty [p_R(x)\{& p_H(x)u_1(W_1 - \bar{p}_H H - \Pi - x + H + I(x)) \\ & + (1-p_H(x))u_1(W_1 - \bar{p}_H H - \Pi - x + I(x))\} \\ + (1-p_R(x))\{& p_H(x)u_1(W_1 - \bar{p}_H H - \Pi - x + H) \\ & + (1-p_H(x))u_1(W_1 - \bar{p}_H H - \Pi - x)\}]\, f(x)dx \end{aligned}$$

s.t.

(6) $\Pi \ge \int_0^\infty p_R(x)I(x)\, f(x)dx,$

(7) $H \ge 0,$

(8) $I(x) \ge 0 \quad \forall x.$

and

(9) $I(0) = 0.$

Let Ω denote the Lagrange multiplier for condition (6). To simplify expressions, we define:

$$
\begin{aligned}
W_P^{H,I}(x) &:= W_1 - \overline{p}_H H - \Pi - x + H + I(x) \\
W_P^{nH,I}(x) &:= W_1 - \overline{p}_H H - \Pi - x + I(x) \\
W_P^{H,nI}(x) &:= W_1 - \overline{p}_H H - \Pi - x + H \\
W_P^{nH,nI}(x) &:= W_1 - \overline{p}_H H - \Pi - x.
\end{aligned} \tag{10}
$$

In the following, characteristics of optimal reinsurance contracts will be derived. Obviously, the optimal indemnity is not unique for levels of loss with $p_R(x) = 0$, but could be chosen arbitrarily, since according to the assumptions reinsurance coverage would not have an effect in these areas of the loss distribution. Therefore, consider the optimal indemnity function $I^*(x)$ for x with $p_R(x) > 0$. The Kuhn/Tucker conditions[16] give

$$p_H(x)u_1'(W_P^{H,I}(x)) + (1 - p_H(x))u_1'(W_P^{nH,I}(x)) = \Omega \tag{11}$$

for x with $I^*(x) > 0$, and

$$
\begin{aligned}
&p_H(x)u_1'(W_P^{H,I}(x)) + (1 - p_H(x))u_1'(W_P^{nH,I}(x)) \\
&= p_H(x)u_1'(W_P^{H,nI}(x)) + (1 - p_H(x))u_1'(W_P^{nH,nI}(x)) \le \Omega,
\end{aligned} \tag{12}
$$

if $I^*(x) = 0$.

As mentioned earlier, the first problem to be tackled here is the way in which credit risk affects reinsurance contracting when index-linked coverage is not available, or respectively, when it is not optimal to buy this kind of coverage. Thus, assume for a moment that $H = 0$, such that (11) and (12) simplify to

$$u_1'(W_1 - \Pi - x + I^*(x)) = \Omega \tag{11'}$$

and

[16] See, e.g., *Chiang* (1984), pp. 722-731.

(12') $\qquad u_1'(W_1 - \Pi - x) \le \Omega.$.

The left hand side in (12') increases in x. Therefore, if for some $\hat{x}$ we have equality in (12'), for any $x > \hat{x}$ the optimal indemnity is positive.

Furthermore, from (11') one can see that the marginal utility is constant for all x with $I^*(x) > 0$,

(13) $$\frac{dI^*}{dx} = 1 \quad \text{when} \quad I^*(x) > 0.$$

Thus, the optimal indemnity function has the following characteristic:

(14) $$I^*(x) = \max(0, x - \hat{x})$$

with $\hat{x} \ge 0$. Up to $\hat{x}$, the *retention* or *deductible*, there is no payment from the optimal reinsurance contract; losses exceeding this amount are covered, but $\hat{x}$ is deduced.

The question remains whether the solution simplifies to $\hat{x} = 0$. This is particularly interesting, as an important result from the theory of insurance demand says that for actuarially "fair" premiums, i.e., if premiums equal the expected losses, and under the assumption of complete information, an insured (here: the primary) would choose complete coverage.[17] According to the following proposition, this result does remain true when credit risk is introduced to the analysis:[18]

Proposition 1:

If $H = 0$*, the optimal reinsurance indemnity function has the form*

$I^*(x) = \max(0, x - \hat{x})$*, with*

(15) $$\hat{x} > 0 \quad \Leftrightarrow \quad \overline{p}_R < 1,$$

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[17] See, e.g., *Raviv* (1979) and among many others *Borch* (1960), *Arrow* (1963), *Borch* (1976).

[18] *Cummins* and *Mahul* (2000) derive a similar result for their model. However, they do not make a statement about whether the solution simplifies to $\hat{x} = 0$, if reinsurance premiums are "fair". For the case of a two-point loss distribution *Doherty* and *Schlesinger* (1990) show that for "fair" premiums incomplete insurance is optimal, if – as has been implicitly assumed here – insurance default means that no coverage at all would be available. They also consider the situation that coverage would be partially available in case of bankruptcy, and show that in this setting other results and even more than complete insurance are possible.

where $\overline{p}_R := \int_0^\infty p_R(x) f(x) dx$.

Proof: See appendix.

The optimal retention is positive if and only if default risk exists. Complete reinsurance is purchased only if, with probability one, the coverage would be available when losses occur.

The problem gets more complicated when index-linked coverage is purchased as part of the optimal solution. General statements concerning the sign of the first derivative of the expression on the left hand side in (12) can only be derived under additional assumptions. Therefore, one cannot conclude that coverage for losses exceeding the deductible is strictly positive. However, the latter can be stated for small loss values with $p_H(x) = 0$.

Differentiating the function implicitly defined by (11) gives additional information about the optimal indemnity scheme:

(16) $$\frac{dI^*}{dx} = 1 - \frac{p'_H(x)\{u'_1(W_P^{H,I}(x)) - u'_1(W_P^{nH,I}(x))\}}{p_H(x)u''_1(W_P^{H,I}(x)) + (1-p_H(x))u''_1(W_P^{nH,I}(x))} \leq 1.$$

In particular, the net coverage

(17) $$N^*(x) := I^*(x) - x$$

decreases in *x*.

The indemnity function can be decreasing. This is consistent with the above-mentioned result that the optimal risk management mix might include a reinsurance contract which leads to alternating loss areas with, in turn, positive indemnity and no coverage.

The results for the case $H > 0$ are summarized in Proposition 2:

Proposition 2:

If $H > 0$, the optimal reinsurance contract has the following properties:

a) *If $I^*(x) > 0$, then:* $\frac{dI^*}{dx} < 1 \Leftrightarrow p'_H(x) > 0$

b) $p_H(x) = 0 \Rightarrow I^*(x) = \max(0, x - \hat{x}), \hat{x} \geq 0$.

These characteristics are quite intuitive. Obviously, index-linked coverage substitutes reinsurance primarily for higher levels of the loss. The slope of the indemnity function depends on how suitable the index-linked coverage is as a reinsurance substitute for the loss range in question. Where the increase of the (contingent) expected value of coverage from the index-linked product is zero, a marginal increase of losses is entirely covered by reinsurance. But whenever $p'_H(x) > 0$, a marginal increase in losses would not be fully indemnified. As can be seen from (16), dI^*/dx is smaller the greater the impact of a marginal loss increase on the suitability of the index product.

After these considerations it has yet to be asked under which conditions index-linked coverage is actually purchased at all. The following proposition provides results with respect to this question.

Proposition 3:

Assume $P\{p_H(\widetilde{X}) \neq \overline{p}_H\} > 0$. *Then the optimal risk management mix fulfills*

(18) $$H = 0 \quad \Leftrightarrow \quad \overline{p}_R = 1.$$

Proof: See appendix.

If no credit risk exists at all ($\overline{p}_R = 1$), a primary has no reason to purchase an index-linked product (under the assumptions of this paper), since risk can be transferred to the reinsurer at no cost. So, complete reinsurance ($I^*(x) = x$) and $H = 0$ would be chosen.

If, however, the reinsurance product carries default risk, and if the index product is in principle a useful hedging tool ($P\{p_H(X) \neq \overline{p}_H\} > 0$), the latter would in any case be purchased. In other words: Risk securitization, based upon an exogenous correlated trigger, improves the risk management mix in the presence of reinsurance default risk.

4 Conclusion

Credit risk (default risk), i.e. – in our context – the threat that payments might not be available even though contractually due when a loss occurs, is a characteristic problem in catastrophe insurance or reinsurance markets. Particularly reinsurance companies with a locally concentrated book of business are subject to a significant risk of bankruptcy after major catastrophes, due to the potential of loss accumulation. Risk securitization on the other hand, for example through the issuance of cat bonds, can be designed such that default risk is entirely avoided.

The possibility of reducing or avoiding default risk as a feature of typical risk securitization transactions provides an additional economic explanation for the demand for these tools. The analysis concentrated on securitization transactions using exogenous triggers, as, for example, market indexes or parametric triggers. Therefore, in the framework considered here, the risk management mix organizes the trade-off between default risk and basis risk.

Before the optimal risk-management mix was studied, it was shown how credit risk affects the structure of an optimal reinsurance contract when reinsurance is used solely: Even though the reinsurance product was assumed to be available at actuarially "fair" rates, which would imply complete coverage in a setting without credit risk, the optimal contract in the presence of credit risk entails a deductible.

Concerning the optimum for the case where both risk management tools are available, it was shown, consistent with the findings of *Nell* and *Richter* (2002), that an index-linked product primarily replaces reinsurance for high levels of the loss. The use of index-linked coverage affects the slope of the indemnity function. Marginally complete reinsurance coverage is possible only for loss levels with a conditional trigger probability of either zero or one. Furthermore it was shown that in the presence of credit risk, index-linked coverage is used in any case, given it has some quality as a hedging tool.

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Appendix

Proposition 1:

If $H = 0$ *, the optimal reinsurance indemnity function has the form*

$I^*(x) = \max(0, x - \hat{x})$ *, with*

$$(15) \qquad \hat{x} > 0 \quad \Leftrightarrow \quad \overline{p}_R < 1,$$

where $\overline{p}_R := \int_0^\infty p_R(x) f(x)dx$.

Proof:

$I^*(x) = \max(0, x - \hat{x})$ has already been proven. Therefore, under the conditions of the proposition the optimization problem can be stated as follows:

$$(19) \qquad \max_{\hat{x}} \; U(\hat{x}) = \int_0^{\hat{x}} u_1(W_1 - \Pi(\hat{x}) - x) f(x)dx + \int_{\hat{x}}^\infty [p_R(x)u_1(W_1 - \Pi(\hat{x}) - \hat{x}) + (1 - p_R(x))u_1(W_1 - \Pi(\hat{x}) - x)] f(x)dx$$

with

$$(20) \qquad \Pi(\hat{x}) = \int_{\hat{x}}^\infty p_R(x)(x - \hat{x}) \; f(x)dx.$$

One gets

$$(21) \qquad \Pi'(\hat{x}) = -\int_{\hat{x}}^\infty p_R(x) \; f(x)dx,$$

and

$$U'(\hat{x}) = -\Pi'(\hat{x})\int_0^{\hat{x}} u_1'(W_1 - \Pi(\hat{x}) - x) f(x)dx$$

(22) $$-(\Pi'(\hat{x})+1)\int_{\hat{x}}^{\infty} p_R(x) u_1'(W_1 - \Pi(\hat{x}) - \hat{x}) f(x)dx$$

$$-\Pi'(\hat{x})\int_{\hat{x}}^{\infty} (1 - p_R(x)) u_1'(W_1 - \Pi(\hat{x}) - x) f(x)dx.$$

If $1 - \overline{p}_R = \int_0^{\infty} (1 - p_R(x)) f(x)dx > 0,$ then

(23) $$\begin{aligned} U'(0) &> -\Pi'(0)\int_0^{\infty} u_1'(W_1 - \Pi(0)) f(x)dx - \int_0^{\infty} p_R(x) u_1'(W_1 - \Pi(0)) f(x)dx \\ &= -u_1'(W_1 - \Pi(0))\int_0^{\infty}\left(p_R(x) - \int_0^{\infty} p_R(x) f(x)dx \right) f(x)dx = 0, \end{aligned}$$

such that „$\Leftarrow$" is true. Conversely, $1 - \overline{p}_R = 0$ implies $U'(0) = 0$.

QED

Proposition 3:

Assume $P\{p_H(\widetilde{X}) \neq \overline{p}_H\} > 0$. *Then the optimal risk management mix fulfills*

(18) $\quad H = 0 \quad \Leftrightarrow \quad \overline{p}_R = 1$.

Proof:

(24) $$\begin{aligned} \frac{\partial U}{\partial H} = \int_0^{\infty} [& p_R(x)\{p_H(x)(1 - \overline{p}_H) u_1'(W_P^{H,I}(x)) - (1 - p_H(x))\overline{p}_H u_1'(W_P^{nH,I}(x))\} \\ & + (1 - p_R(x))\{p_H(x)(1 - \overline{p}_H) u_1'(W_P^{H,nI}(x)) \\ & \qquad - (1 - p_H(x))\overline{p}_H u_1'(W_P^{nH,nI}(x))\}] f(x)dx. \end{aligned}$$

Consider

$$(25)\qquad \left.\frac{\partial U}{\partial H}\right|_{H=0} = \int_0^\infty [p_R(x)\{p_H(x)(1-\overline{p}_H)-(1-p_H(x))\overline{p}_H\}u_1'(W_P^{H,I}(x))$$
$$+(1-p_R(x))\{p_H(x)(1-\overline{p}_H)-(1-p_H(x))\overline{p}_H\}u_1'(W_P^{H,nI}(x))]f(x)dx$$

$$=\int_0^\infty (p_H(x)-\overline{p}_H)[p_R(x)u_1'(W_P^{H,I}(x))+(1-p_R(x))u_1'(W_P^{H,nI}(x))]f(x)dx.$$

Obviously, this expression equals zero, if $\overline{p}_R = 1$ and therefore $P\{p_R(x)=1\}=1$. This is due to the fact that, according to Proposition 1, $u'(W_P^{H,I}(x))$ is constant for $H=0$. This proves „$\Leftarrow$".

To finish the proof, one needs only to show that the expression in (25) is positive, if $\overline{p}_R < 1$. For this purpose, we prove that the term in square brackets in the last expression in (25) is increasing (and strictly increasing in an area with positive probability mass) in x (note that, if this is the case, larger values of $p_H(x)$ are weighed more heavily in (25), such that the expected value in total is positive).

Let

$$(26)\qquad h(x) := p_R(x)u_1'(W_P^{H,I}(x))+(1-p_R(x))u_1'(W_P^{H,nI}(x)).$$

One gets

$$(27)\qquad h'(x) := p_R'(x)u_1'(W_P^{H,I}(x))+p_R(x)u_1''(W_P^{H,I}(x))(\frac{dI^*}{dx}-1)$$
$$-p_R'(x)u_1'(W_P^{H,nI}(x))-(1-p_R(x))u_1''(W_P^{H,nI}(x)).$$

According to Proposition 1, the optimal contract for $C=0$ is characterized by $I^*(x)=\max(0,x-\hat{x})$ with $\hat{x}>0$. For $x<\hat{x}$ (27) simplifies to

$-u_1''(W_P^{H,I}(x)) = -u_1''(W_P^{H,nI}(x)) > 0.$

For $x>\hat{x}$ ($\Rightarrow I^*(x)>0$, $dI^*/dx=1$)), because of $u_1'(W_P^{H,I}(x)) < u_1'(W_P^{H,nI}(x))$ and $dI^*/dx-1=0$, (27) is non-negative and positive at least where $p_R(x)<1$.

QED