# Relationship Between Deductibles and Expected Payouts for Insurance Policies

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## ABSTRACT

Most insurance policies provide for a “deductible” clause. The level of the deductible is of interest both to the insurer and the insured. In this paper, we examine the effects of the deductible on the expected payout of an insurance policy. The results provide guidelines to the insured in choosing an appropriate level of the deductible and to the insurer, the level of discount in premium to be offered for varying levels of deductibles.

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## 1. INTRODUCTION:

Most insurance policies provide for a "deductible" clause; the insurer pays a loss only when the "deductible" specification is satisfied. Typically a large number of claims are small, and since the administrative costs of settling a claim can be quite high, the ability to eliminate small claims from consideration as a result of a deductible can result in significant reductions in the cost of doing business for the insurer. This, in turn, will generally lead to lower premiums for the insured with a deductible clause than without the deductible clause. Arrow (1971, 1974) and Raviv (1979) agree that the insured wants to have less than full insurance because the insurance is not actuarially fair. Furthermore, since a deductible would require the insurer to bear part of a loss, Shavell (1979) ties the deductible with the possibility of moral hazard, emphasizing that with a deductible, the insured finds it worthwhile to make an effort to prevent and/or control a loss.

Thus, a deductible is advantageous to both the insurer and the insured; the cost of doing business is lowered for the insurer while the insured pays a lower premium. The reduction in the premium, typically, would not be directly proportional to the size of the deductible but would be on a sliding scale that would reflect the decreasing probability of larger losses.

The fact that many insurance agreements stipulate a deductible has been the subject of many studies. Schlesinger (1981) examines the optimal deductible in insurance contracts and shows that it is related to the insured's degree of risk aversion. Doherty and Schlesinger (1983) examine the optimal deductible in insurance contracts when the initial wealth is random. Demers and Demers (1991) examine the impact of increasing risk on the optimal deductible for a risk averse insured and a risk neutral insurer. Barniv, Schroath, and Spivak (1999) examine the effect of wealth on the demand for deductibles, both theoretically and empirically, for the case of flood insurance. The authors concentrate on the association between the deductible selected and the wealth of the insured.

None of the studies cited above have examined the effect of varying levels of the deductible on the expected payout of the insurer. However, this aspect will be of interest to both the insurer and the insured. For the insurer, this information can be of value in deciding the reduction in the premium to be offered. For the insured, the information can aid in choosing an appropriate level of deductible.

Deo (1996) has studied this problem for the petro-chemical industry in India. In general, the losses in the petro-chemical industry tend to be extremely high. She found that the Log-Normal distribution describes this loss experience quite well. In this paper, we are interested in situations where small to medium losses are more common. In such situations, the loss experience can be modeled by exponential distributions. Hence, in our investigations, we assume that the loss for a claim is exponentially distributed.

## 2. DEDUCTIBLES:

There are many types of deductibles that are prevalent in the insurance field. In this paper, we consider the following three types of deductibles that are among the most common types of deductibles.

(i) A straight deductible: A straight deductible is expressed as a specified amount $D$. Any loss that is less than the deductible amount $D$ is not paid; when the loss exceeds $D$, the insurer pays (loss amount -$D$). This type of straight deductible is typical of auto insurance policies.

(ii) Percentage deductible: A percentage deductible is expressed as a percentage of the loss amount. A 10% deductible would mean that the insurer would pay 90% of the loss amount. This type of deductible is typical of health care policies.

(iii) A combination of straight and percentage deductible: Here the deductible is a fixed amount $D$ and a fraction of the loss exceeding $D$. This type of deductible is again common of health care policies.

## 3. NOTATION AND MODEL:

The following notation will be used in the subsequent discussions.

$L$ = loss on a policy,

$D$ = fixed deductible amount,

$f$ = deductible fraction,

$P$ = payout on a policy,

$f(\bullet)$ = probability density function of $L$, and

$F(\bullet)$ = cumulative distribution function of $L$.

We assume that

$f(l) = \lambda\, e^{-\lambda l}, l \geq 0$

where

$\frac{1}{\lambda} = E(L) =$ mean loss on a policy.

Hence $\quad F(x) = P(L \leq x) = 1 - e^{-\lambda x}$.

4. <u>THE STRAIGHT DEDUCTIBLE</u>:

Here, $\quad P = 0 \quad$ if $L \leq D$

$\quad\quad = L\text{-}D \quad$ if $L > D$.

$$E(P) = \int_D^{\infty} (l - D)\lambda e^{-\lambda l}\, dl = \frac{1}{\lambda} \cdot e^{-\lambda D}$$

Observe that for $D = 0$, $E(P) = \frac{1}{\lambda} = E(L)$, as it should be.

Also, as $D$ increases, $E(P)$ decreases, as it should.

Savings in the expected payout as a result of the deductible $= \frac{1}{\lambda} - \frac{1}{\lambda} \cdot e^{-\lambda D}$

$= \frac{1}{\lambda}\, F(D)$

Hence the proportionate savings in the expected payout as a result of the straight deductible D is F(D). The insurer can use this proportion as a guide to determine the discount in premium to be offered for varying levels of D.

It should be noted that as D increases, the proportionate savings in the expected payout by the insurer increases and consequently, the savings in the premium for the insured should also increase. However, $F(D)/D \to 0$ as D increases. Hence, the marginal benefit to the insured of increasing the deductible level D would be decreasing and the insured should take this into consideration in choosing an appropriate level of $D$.

5. PROPORTIONATE DEDUCTIBLE:

Here, the deductible $D = f \cdot L$ where $0 \leq f \leq$

Hence, $P = (1-f)L$

$$E(P) = (1-f) \cdot E(L) = {(1-f)}/{\lambda}$$

Savings in the expected payout $= {f}/{\lambda}$

Proportionate savings $= f$

Again, the insurer can use the fraction $f$ as a guide to the discount to be offered in the premium. In the case of proportionate deductible, note that the "average marginal benefit" to the insured (in the form of proportionate savings in the premium) would be:

$$f/E(D) = f/E(f \cdot L) = \lambda$$

Thus, the average marginal benefit is a constant, independent of the deductible fraction $f$. This situation is thus different from the straight deductible case.

6. DEDUCTIBLE A COMBINATION OF STRAIGHT DEDUCTIBLE AND PROPORTIONATE DEDUCTIBLE:

Here the deductible = a fixed amount $D$ + a fraction f of any loss in excess of $D$.

Thus, $P = 0$ if $L \leq D$

$= (1-f)(L-D)$ if $L>D$

$$\text{Hence} \quad E(P) = (1-f) \int_D^{\infty} (l-D)\, f(l)\, dl$$

$$= (1-f) \cdot {1}/{\lambda} \cdot e^{-\lambda D}$$

Savings in expected payout as a result of the deductible $= {1}/{\lambda} \left[1-(1-f)e^{-\lambda D}\right]$

Proportionate savings $= 1-(1-f)e^{-\lambda D}$

Observe that, for a fixed *f*, $(1\text{-}f)\,e^{-\lambda D}$ decreases exponentially as *D* increases and, hence, for a fixed *f*, the proportionate savings increase exponentially as D increases.

On the other hand, for a fixed *D*, $(1\text{-}f)\,e^{-\lambda D}$ decreases linearly as f increases and hence, for a fixed *D*, the proportionate savings increase linearly with *f*.

Hence, given a choice between a higher value of *D* and a higher value of *f*, a customer should choose a higher value of *D* for a greater savings in the expected payout, leading to a greater reduction in the premiums.

7. ESTIMATION OF λ:

We have $E(L) = 1/\lambda$.

Thus, we need to estimate the mean loss per policy. However, for the cases of the straight deductible and the combination deductible, we observe (record) *P*, the payout on a claim, only when *L* exceeds *D*.

For the case of straight deductible, we have $E(P) = 1/\lambda \cdot e^{-\lambda D}$

We can obtain an estimate of $\lambda$ by solving the equation

$$\overline{P} = 1/\lambda \cdot e^{-\lambda D}$$

Where $\overline{P}$ is the mean of the observed payouts.

For the case of the combination deductible, since $E(P) = (1\text{-}f) \cdot 1/\lambda \cdot e^{-\lambda D}$

We can obtain an estimate of $\lambda$ by solving the equation $\overline{P} = (1-f) \cdot 1/\lambda \cdot e^{-\lambda D}$

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## TABLE 1 - Values of (f(D))/D

$$\boxed{f(D) = 1 - e^{-\lambda D}}$$

| $\lambda$ / $D$ | **50** | **100** | **150** | **200** | **250** | **500** |
|---|---|---|---|---|---|---|
| **0.0010** | 0.000975 | 0.000952 | 0.000929 | 0.000906 | 0.000885 | 0.000787 |
| **0.0005** | 0.000494 | 0.000488 | 0.000482 | 0.000476 | 0.000470 | 0.000442 |
| **0.0002** | 0.000199 | 0.000198 | 0.000197 | 0.000196 | 0.000195 | 0.000190 |
| **0.0001** | 0.000100 | 0.000100 | 0.000099 | 0.000099 | 0.000099 | 0.000098 |

**Values of Pr S, the proportionate savings, for $\lambda$=0.001**

**(Mean Loss = 1000)**

| | | D | | | | | |
|---|---|---|---|---|---|---|---|
| | | **50** | **100** | **150** | **200** | **250** | **500** |
| **f =** | .10 | 0.1439 | 0.1856 | 0.2254 | 0.2631 | 0.2991 | 0.4541 |
| | .15 | 0.1915 | 0.2309 | 0.2684 | 0.3041 | 0.3380 | 0.4844 |
| | .20 | 0.2390 | 0.2761 | 0.3114 | 0.3450 | 0.3770 | 0.5148 |
| | .25 | 0.2866 | 0.3214 | 0.3545 | 0.3860 | 0.4159 | 0.5451 |

**Table 2**

**Values of Pr S, the proportionate savings, for $\lambda$ = 0.0005**

**(Mean Loss = 2000)**

| | | D | | | | | |
|---|---|---|---|---|---|---|---|
| | | **50** | **100** | **150** | **200** | **250** | **500** |
| **f =** | .10 | 0.1222 | 0.1439 | 0.1650 | 0.1856 | 0.2058 | 0.2991 |
| | .15 | 0.1710 | 0.1915 | 0.2114 | 0.2309 | 0.2499 | 0.3380 |
| | .20 | 0.2198 | 0.2390 | 0.2578 | 0.2761 | 0.2940 | 0.3770 |
| | .25 | 0.2685 | 0.2866 | 0.3042 | 0.3214 | 0.3381 | 0.4159 |

**Table 3**

**Values of Pr S, the proportionate savings, for $\lambda$ = 0.0002**

**(Mean Loss = 5000)**

| | | D | | | | | |
|---|---|---|---|---|---|---|---|
| | | **50** | **100** | **150** | **200** | **250** | **500** |
| **f =** | .10 | 0.1090 | 0.1178 | 0.1266 | 0.1353 | 0.1439 | 0.1856 |
| | .15 | 0.1585 | 0.1668 | 0.1751 | 0.1833 | 0.1915 | 0.2309 |
| | .20 | 0.2080 | 0.2158 | 0.2236 | 0.2314 | 0.2390 | 0.2761 |
| | .25 | 0.2575 | 0.2649 | 0.2722 | 0.2794 | 0.2866 | 0.3214 |

**Table 4**

**Values of Pr S, the proportionate savings, for $\lambda = 0.0001$**

**(Mean Loss = 10,000)**

| | | **D** | | | | | |
|---|---|---|---|---|---|---|---|
| | | **50** | **100** | **150** | **200** | **250** | **500** |
| **f =** | .10 | 0.1045 | 0.1090 | 0.1134 | 0.1178 | 0.1222 | 0.1439 |
| | .15 | 0.1542 | 0.1585 | 0.1627 | 0.1668 | 0.1710 | 0.1915 |
| | .20 | 0.2040 | 0.2080 | 0.2119 | 0.2158 | 0.2198 | 0.2390 |
| | .25 | 0.2537 | 0.2575 | 0.2612 | 0.2649 | 0.2685 | 0.2866 |

**Table 5**